Alabama State Department of Education

SECC-AL State Plan for Progress in Mathematics

The contents of this plan were developed under a grant from the Department of Education. However, those contents do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

June 2018

RESEARCH & DEVELOPMENT
ALABAMA STATE DEPARTMENT OF EDUCATION

SOUTHEAST Comprehensive Center
at American Institutes for Research

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Contents

Introduction—Background

For the past 25 years, student achievement in the state of Alabama has grown at a respectable rate in both fourth- and eighth-grade mathematics, as measured by the National Assessment of Educational Progress (NAEP). That growth, however, has not matched the growth rate of the rest of the nation, causing Alabama to be ranked last or next to last for NAEP scores in mathematics in 2015. While the national average of educational improvement over the past quarter century has been +2.7 years of ability for fourth-grade students, Alabama only averaged +2.3 years of ability. Similarly, Alabama's +1.4 years of ability gains for eighth-grade mathematics since the 1990s falls far short of the national average of +1.9 years of ability. Alabama's students have not improved at the same pace as students in other states.

Leaders at the Alabama State Department of Education (ALSDE) were not satisfied with the rate of growth in mathematics scores continuing to fall further behind the national average. In January 2017, a group of highly experienced experts in the field of mathematics convened, and this group became the Alabama Mathematics Strategic Planning Committee. Stakeholder groups from around the state submitted nominees to serve on the committee. The 26-member committee represented each of the eight school districts and was composed of members with a variety of mathematics experience, including K–12 mathematics teachers, district superintendents, school board members, mathematics specialists and coaches, mathematics teacher educators from higher education, mathematics instructors from community colleges, business community leaders, and the state mathematics director from ALSDE. The committee's charge was to develop recommendations to improve mathematics education for all students in Alabama without reservations regarding tradition, current practice, or financial implications.

Before the committee was to begin meeting in February 2017, ALSDE leaders wished to have a variety of vetted research studies and papers available for the members to review to help guide their recommendations. Work began on compiling a list of research studies for use by the committee members. Publicly available literature from the What Works Clearinghouse (WWC) and the Education Resources Information Center (ERIC) was uploaded by ALSDE staff to the committee's common SharePoint site. Although the research that staff found was peer reviewed, the reports were not quite specific enough to answer the important question: What should teachers know and be able to do to effectively teach mathematics?

State leadership believed that ALSDE staff might benefit from bringing in another organization to help compile research around mathematics. The Southeast Comprehensive Center (SECC) at American Institutes for Research (AIR) was contacted to assist with finding timely, rigorous, and impactful research. Initially, ALSDE had wanted an overview of some exceptional mathematics programs or strategies, including the context in which those programs or strategies were implemented and ways in which that context played into their success. The results could then inform the committee's work on improving mathematics instruction in Alabama. The committee finished its work and submitted recommendations to the superintendent ahead of schedule; therefore, ALSDE staff and SECC began work on a larger, more in-depth literature review that would address three specific research questions (RQ):

1. What specific knowledge do students need to have to make the transitions to higher level mathematics?[1]
2. What core knowledge do teachers need to have about mathematics in order to have students successfully make those critical transitions to higher level mathematics?
3. What are best practices that teachers should use to teach mathematics—specifically, concepts related to higher level mathematics?

Methodology and Screening Process

The project team created a systematic approach to identify and screen relevant literature. The process involved the development of research questions, key terms, and the process for cataloguing literature. (For more details on the process, see Appendix A.)

Themes

The literature review included three rounds of review. In the initial round, project members searched for literature based on key words and terms associated with the research questions. For the second round of review, the team screened the articles to identify the strength of the literature, examining the type of study (e.g., conceptual or theoretical framework, research report, the methods employed, and its key findings). In the final round of review, the team reviewed the articles again and focused on emergent themes across the literature. Team members also agreed on protocol for conducting a thematic synthesis. (See Appendix B.)

The findings from the literature review are organized into themes by research question. Within each theme are references to relevant literature, which include conceptual or theoretical frameworks, and empirical evidence. Themes include the following:

[1] For the purposes of this literature review, *higher level mathematics* will be defined as "high school mathematics courses beginning with Algebra I."

- *Research Question 1. Student knowledge themes:* fractions, proportional reasoning, and the role of mathematical language and symbolism.
- *Research Question 2. Teacher knowledge themes:* fractions, proportional reasoning, mathematical language, teacher and student use of mathematical knowledge, mathematical knowledge for teaching, and teacher content knowledge.
- *Research Question 3. Instructional practice themes:* explicit and systematic instruction, instruction on solving word problems, and facilitation of meaningful mathematical discourse.

Research Question 1. What Specific Knowledge Do Students Need to Have to Make the Transitions to Higher Level Mathematics?

Mathematics is a complex discipline replete with concepts, processes, and properties, as well as explicit language and symbolism used to describe and represent them. Mathematics also is a discipline in which the components mentioned above are often inextricably connected and interdependent, and in which lack of knowledge in one creates difficulties in the learning of another. Current literature from the research base was reviewed to identify types of mathematical knowledge necessary for students to successfully progress from mathematics in early grades to the more complex mathematics at the middle- and high-school levels. The literature search yielded a multitude of concepts; however, themes emerged that brought out the topics that were the most critical in students' successful transition to and learning of more complex mathematical concepts. These themes include the concepts of fractions and proportional reasoning, as well as the role of mathematical language and symbolism.

Understanding Fractions Is a Key to Success

A major theme that emerged as a key prerequisite for future mathematics success was that of fractions. Forty articles were selected for the literature review regarding problem topics for students. Of those, 23 were deemed relevant to the literature review. Further investigation revealed that 15 of those 23 dealt with fractions. According to Hansen et al. (2015), "fraction knowledge in middle school accounts for much of the gains students make in mathematics achievement" (p. 35). Shin and Bryant (2015) add that conceptual understanding of fractions is one of the critical foundations of algebra and is therefore an essential building block for successfully advancing in secondary mathematics. Findings by Siegler et al. (2012) also support the assertion that mastery of fractions is necessary for the understanding of algebra and other aspects of high school mathematics. Because of the importance of competence with fractions as part of the learning progression for algebra, poor performance outcomes for fractions are especially alarming (Shin & Bryant). Mastery of fraction concepts and operations with fractions are critical, and this is a major theme that must be addressed (National Mathematics Advisory Panel, 2008). Teachers recognize the seriousness of the problem: Siegler and Lortie-Forgues' (2015) survey of a representative sample of U.S. high school algebra teachers rated knowledge of fractions as "one of the two largest weaknesses in their students' preparation for their course, from among 15 topics in mathematics" (p. 909).

Most American adults experienced the difficulties with understanding fractions. Charalambous and Pitta-Pantazi (2007) indicates that fractions are among the most complex mathematical concepts that students encounter and that learning fractions is probably one of the most serious obstacles to their mathematical maturation. The concept of fractions is a mathematical topic consisting of multiple components and issues. Consequently, there are many potential sources of students' difficulties with fractions. According to Charalambous and Pitta-Pantazi, "Although during the past 3 decades several factors have been identified as contributing to students' difficulties in learning fractions, researchers and scholars agree that one of the predominant factors contributing to the complexities of teaching and learning fractions lies in the fact that fractions comprise a multifaceted construct" (p. 293).

Fundamental concepts in mathematics typically serve as the building blocks for other, more complex topics. However, there are instances in which knowledge of one topic can hinder the learning of another. Such is the case with fractions and operations with fractions. One issue is referred to as "whole number bias," which refers to students' tendency to apply a single-unit counting scheme to fractions which then causes difficulty in perceiving whole numbers as decomposable units (Ni & Zhou, 2005). Fractions are often rejected as numbers by students because fractions are not part of the counting sequence and do not fit into their construct of place value (Charalambous & Pitta-Pantazi, 2007). Student confusion results in initial beliefs such as 1/3 being greater than 1/2 because 3 is greater than 2. Ni and Zhou assert that "instruction on fraction numbers in many classrooms neglects to pay adequate attention to the conceptual conflict that develops and the conceptual restructuring that is required for the transformation to take place from the concept of whole numbers to that of fraction numbers in children" (p. 35).

The mathematics that students learn initially in early elementary school involves only the use of whole numbers. Thus, students relate fractions to their knowledge of whole numbers. Part of the difficulty in the transition to rational numbers is that the rules or patterns to which students are accustomed do not always apply to fractions. For example, Booth and Newton (2012) note that, when adding fractions, a common error is to treat the components of fractions as whole numbers, thus adding numerators together as well as denominators, and that error continues to be persistent even when students have been warned about it. Operations with whole numbers also have instilled a belief that multiplication yields products greater than either factor and that dividing yields quotients smaller than the dividend. Incorrect answers will often seem more reasonable than correct ones, and this results in situations in which 12/5 is more plausible than the correct answer of 12/25 for the multiplication problem 3/5 x 4/5 (Siegler & Lortie-Forgues, 2015).

According to Booth and Newton (2012), "numerical development involves a gradual expanding and refining of the definition of number, which includes learning what features hold for all numbers as opposed to particular types of numbers" (p. 248). Part of this numerical maturation involves understanding the magnitude of numbers. Booth and Newton suggest that a lack of understanding of the magnitudes, or relative sizes, of fractions is rampant among school-aged children. State and national tests typically reveal that American students have difficulty determining the correct order or sequence of proper fractions (fractions between 0 and 1) or tasks such as finding a decimal between two other decimals, such as a number between .04 and .05. For example, Siegler and Lortie-Forgues (2015) found that, on a recent National Assessment of Educational Progress (NAEP), only 50% of eighth graders correctly ordered from smallest to largest 2/7, 5/9, and 1/12.

A hindrance for understanding fraction magnitude is that instruction is often focused on fractions from the part–whole interpretation at the expense of teaching fractions as a measure or as a real number represented on a number line. Research suggests (Booth & Newton, 2012) that number line representation is especially important in the early grades, which is consistent with the Common Core Standards' emphasis regarding the use of number lines for fraction learning as early as third grade. Hansen et al. (2015) assert that it is critical for students to comprehend that a common property uniting fractions and whole numbers is the fact that all real numbers have magnitudes that can be ordered along a continuous number line. In addition, according to Hansen et al., "students who develop an understanding that each real number, including each fraction, is assigned a specific location on the number line have an advantage in learning not only fractions but also algebraic concepts" (p. 46). Booth and Newton (2012) further reinforce that notion with the results from their study, which suggested that knowledge of fraction magnitudes is even more important for algebra readiness than whole-number magnitude. With respect to numerical development, it also is imperative that students recognize that each natural number has a unique predecessor and a unique successor and that an infinite number of fractions can be located between any two numbers (Siegler & Lortie-Forgues, 2015).

Having a conceptual understanding of fractions and fraction magnitude also plays a major role with successful computation with fractions. Even with the operation of addition, the simplest of the four operations, students' resistance to accept fractions as numbers leads them to view fractions as two different whole numbers, a misconception that can result in computational errors, such as 1/2 + 1/4 = 2/6 (Charalambous & Pitta-Pantazi, 2007). Facility or fluency with operations using fractions should include, at a minimum, understanding the direction of the outcome of the operation (i.e., the operation results in a larger solution or a smaller solution).

Siegler and Lortie-Forgues (2015) examined this type of misunderstanding of fraction arithmetic, which has been coined as “direction of effects errors” (p. 910). In that study (Siegler & Lortie-Forgues), middle school students and preservice teachers “demonstrated minimal understanding of the magnitudes produced by multiplication and division of fractions” (p. 914). That study found that both sets of participants consistently predicted incorrectly that multiplying two proper fractions would yield a product greater than either factor and that dividing by a proper fraction would result in a quotient smaller than the number being divided.

To help understand the cause of those misconceptions, Siegler and Lortie-Forgues (2015) suggest that participants applied their understanding of whole-number arithmetic to the direction-of-effects task with fractions “not because they were convinced that this was correct but because they did not know what else to do” (p. 915), adding that when people lack understanding, they will resort to default knowledge in such situations. In addition, even if students attain the correct responses in fraction arithmetic, there is no guarantee that they truly understand the process or the magnitude of the result of the operation. Siegler and Lortie-Forgues (2015) note that

> Mastery of fraction arithmetic procedures—the algorithms or rules used to solve mathematical problems—could easily coexist alongside weak or inaccurate conceptual understanding of the procedures—implicit or explicit knowledge of how the procedures work, why they make sense, and how they are related to other procedures and concepts in the domain (p. 910).

According to Shin and Bryant (2015), “lack of conceptual understanding of and computational facility with fractions limits students’ ability to solve more advanced computational problems, including ratios, rates, and proportions, all of which are critical foundational skills for algebra” (p. 375). To the untrained eye, it may be difficult to ascertain the importance of the connection of fractions to algebra. Siegler et al. (2012) indicate that without conceptual understanding of fractions and fraction computation, students cannot estimate or judge the reasonableness of solutions to simple algebraic equations. For example, students weak in fraction expertise will not realize that in an equation such as 1/4 A =1/2 B, that the value of A must be twice that of B, or that in an equation such as 2/3 X = 6, the value of X must be somewhat larger than 6 since the 6 is a portion of a whole larger than 6.

As noted, the concept of fractions is complex because it can have multiple meanings, with many educators typically citing four subconstructs. These include interpretations of a fraction as a part–whole subconstruct, a ratio subconstruct, a division subconstruct, and a measure subconstruct that includes a fraction as a real number and as an interval measure. In a study of student performance on these subconstructs, Charalambous and Pitta-Pantazi (2007) revealed that the best performance was on the part–whole interpretation, and the worst was on fractions as a measure. Most Americans who experienced the traditional system of mathematics education realize the preponderance of the part–whole interpretation of fractions compared with the other subconstructs. Therefore, it seems reasonable for Charalambous and Pitta-Pantazi to assert that “the differences in students’ performance on the fraction subconstructs mirror the imbalance in emphasis placed on them during instruction” (p. 309). This is problematic, because of the foundational role that fractions as ratios and measures play in topics such as algebra and proportional reasoning.

The different subconstructs of fractions reveal that a fraction can essentially be a form of division. In turn, division is the only whole-number operation that can result in fractional answers. Similar to fraction operations such as addition of fractions with unlike denominators, the traditional long division algorithm requires students to integrate several arithmetic operations and remember multiple steps (Hansen et al., 2015). Because of those parallels, Hansen et al. propose that “facility with whole number long division has a uniquely important role in the development of fraction knowledge” (p. 36). The connection between fractions and division is not inherently obvious but is potentially important, making it unfortunate that division concepts are often underemphasized in school (Siegler et al., 2012). Usiskin (2007) asserts that “the realization that fractions represent division and constitute the most common way in which division is represented in algebra has caused a demand for increasing competence with fractions by all those for whom algebra skills are important” (p. 370).

Notably, sixth grade is a key benchmark period for examining competence with fractions, and it is often the last year students receive instruction with an intensive focus on fractions (Hansen et al., 2015). For students to experience success at higher grade levels in mathematics, educators must ensure a solid foundation in both the conceptual understanding and procedural fluency of fractions. Hansen et al. support this assertion by pointing out that, by the end of sixth grade, “students should show proficiency in fraction equivalence and ordering as well as in executing complex fraction arithmetic procedures for all four operations, to provide a foundation for more advanced mathematical topics” (p. 47). As has been noted, key among the critical topics are fractions and division. To emphasize this central theme, analyses of large, nationally representative, longitudinal data sets from the United States by Siegler et al. (2012) revealed that “elementary school students’ knowledge of fractions and of division uniquely predicts those students’ knowledge of algebra and overall mathematics achievement in high school, 5 or 6 years later, even after statistically controlling for other types of mathematical knowledge, general intellectual ability, working memory, and family income and education" (p. 691).

Proportional Reasoning

The subconstruct of a fraction as a ratio provides a bridge to proportional reasoning, another major theme from the research base. Proportional reasoning occupies a critical, pivotal position in the mathematics educational progression. This is a topic that warrants maximum attention and focus, because proportional reasoning is simultaneously the capstone of elementary arithmetic and the cornerstone of much of the mathematics that is to follow (Pelen & Artut, 2016). In addition, "no area of elementary school mathematics is as mathematically rich, cognitively complicated, and difficult to teach as proportionality" (Pelen & Artut, p. 30). The research uncovered for this report strongly suggests that students who do not develop a strong proportional reasoning foundation will in all likelihood encounter difficulties in higher level mathematics.

> **For example:** Students should see 12 as being 4 times larger than 3, as opposed to only seeing 12 as 9 more than 3.

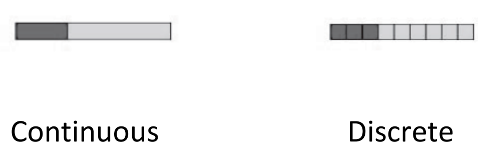
Ratios convey the notion of a comparison between two quantities, so they represent a comparison or relationship rather than a number, which is a new transition for students. Rather than interpreting a fraction as a quantity such as 2/3 of a pizza, students must recognize that 2/3 can represent a relationship such as 2 cats for every 3 dogs. Equivalent ratios are expressed as proportions that connect to previous student experience with equivalent fractions. Students having difficulties with equivalent fractions and fraction magnitudes will be confronted with additional challenges. Proportions symbolize proportional reasoning, which involves multiplicative thinking. Students must not only transition to this perspective but learn to identify and distinguish between additive and multiplicative situations (Van Dooren, Bock, & Verschaffel, 2010).

It is critical that students overcome the aforementioned challenges because "understanding of proportionality is central to mathematics as it is the basis of rational number operations, unit partitioning, and basic algebra and geometry problem solving" (Boyer, Levine, & Huttenlocher, 2008, p. 1478). Proportionality plays a critical role in primary and secondary mathematics because of its wide applicability, but mastery of the required multiplicative reasoning in proportional situations is not easily achieved (Van Dooren et al., 2010). One of the contributing factors is that students have little experience with ratios and proportionality, since these concepts are not typically taught until sixth grade. Students do not have early experiences with proportional reasoning; what limited instruction they receive has little depth and breadth.

Van Dooren et al. assert that students typically learn proportional reasoning with missing-value proportionality problems in which three values are given and a fourth is unknown. Students then utilize the cross-multiplication procedure to solve for the unknown item. According to Van Dooren et al., the missing-value formulation of a proportional-reasoning word problem is probably its most salient feature, and a lot of attention is paid to the development of fluency in solving these problems.

Studies such as that of Van Dooren et al. (2010) have shown that young students can successfully handle simple proportional situations; yet little is done to develop this concept in elementary school. Studies like that of Boyer et al. (2008) have illustrated issues that can be addressed to avoid difficulties in future instruction. That study revealed that students were influenced by the representations of the items being compared. The results from Boyer et al. (2008) revealed that even first-grade students could successfully solve proportional equivalence problems when the choices were all represented with continuous quantities, but they failed with parallel problems when the choices were represented with discrete representations (Figure 1).

Figure 1. Examples of Continuous and Discrete Representations



The findings of Boyer et al. (2008) suggest that "young children go wrong in reasoning about proportions when the knowledge they have acquired about counting to compare set sizes gets in the way of their intuitive, relative visual comparison, proportional reasoning processes" (p. 1488). This is another clear indicator to educators that, although mathematical knowledge is connected and topics often serve as building blocks for other concepts, what is previously learned in mathematics can sometimes hinder future learning. These types of issues must be identified and addressed in order for students to succeed in more complex mathematics.

To become proficient in proportional reasoning, students must become multiplicative thinkers. However, just knowing multiplication facts does not make a student a multiplicative thinker (Hurst & Hurrell, 2016). As noted in the study by Boyer et al. (2008), students' issues on proportional situations stem from their "propensity to compare quantities on the basis of the number of elements in the target quantity rather than on the basis of proportional relations" (p. 1487). As with the central change required when transitioning from natural numbers to rational numbers, students must face a radical shift from additive concepts to multiplicative concepts, which is difficult because of the elementary school focus on comparing quantities on the basis of how much larger or smaller one number is than another (Pelen & Artut, 2016). The multiplicative second-order relationship is difficult for students "because it requires more complicated mental structures than simple multiplication and division" (Pelen & Artut, p. 30). In addition, Van Dooren et al. (2010) assert that "the repeated addition model for multiplication is incomplete, and that a significant qualitative change is required to get from additive to multiplicative thinking" (p. 361).

Consider this: During the same period, a small pig grew from 5 pounds to 10 pounds, and a larger pig grew from 100 to 108 pounds. Which pig grew more? The ambiguous question leaves students with both additive and multiplicative options. The correct response would be "it depends," with students explaining both perspectives. However, if eighth-grade students' only response is that the big pig grew more because 8 pounds is more than 5 pounds, then that student group may have a significant transition problem. Inherent in this context is the probability that such students learned multiplication without the underpinning conceptual understanding based on place value, which is a multiplicative concept.

Hurst and Hurrell (2016) assert that conceptual understanding in concert with procedural fluency and place value knowledge are critical. These authors also indicate that "the development of genuine multiplicative thinking (i.e., more than remembering and recalling number facts) has been hindered through the teaching of procedures at the expense of conceptual understanding" (p. 34). One perspective in this context is that learning multiplication facts and the procedure for multiplication is procedural fluency, but multiplicative thinking is the exercise of conceptual understanding (Hurst & Hurrell).

Van Dooren et al. (2010) indicate that elementary students perform well on multiplicative tasks dealing with one-to-many correspondence such as "every horse has 4 legs, so how many legs do 5 horses have?" because those can be perceived as repeated addition, but those same students falter on tasks where multiplication needs to be conceived and handled differently. Understanding and transitioning to multiplicative thinking as much more than a faster way of doing repeated addition is difficult. However, an equally formidable issue that students face is differentiating between additive and multiplicative situations. Research such as that by Van Dooren et al. revealed several problems with teaching of proportional reasoning that present challenges to students' differentiation of additive and proportional situations.

> **Consider this:** George and José are running laps around a track. They run at equal speeds, but José started later. When José has run 2 laps, George has run 3 laps. When José has run 8 laps, how many has George run?

When students are learning proportional reasoning, this is not a typical task that students are given. Van Dooren et al. (2010) assert that the majority of proportional-reasoning word problems that middle school students encounter are very similar in that they are typically formulated in a missing-value format. The focus for these proportional problems is often the fluent execution of the arithmetic procedures, which in these cases, would be the process of cross-multiplication. Students will use the strategies that have recently been taught. Consequently, middle school students will default to using a multiplicative approach even with problems requiring an additive approach. Modestou and Gagatsis (2007) contend that proportionality is so embedded in middle-school student thought that it "tends to be applied to problem situations without any consideration of the realistic constraints" (p. 76). According to Van Dooren et al.

> Between the initial stage where students over-generalize additive methods and the later stage where they over-generalize proportional methods, students' behavior is not characterized by a correct use of additive and proportional methods. Rather, they switch between methods, not on the basis of the model underlying the problem situation but based on a superficial problem characteristic that should not have any impact on their choice for a solution method, namely whether the numbers that are given in a word problem form integer ratios or not (p. 375).

Pelen and Artut (2016) assert that "proportional reasoning encompasses not only reasoning about the holistic relationship between two rational expressions but wider and more complex spectra of cognitive abilities which include distinguishing proportional and non-proportional situations" (p. 31). The authors suggest that students should be faced with both proportional and additive problems in order to comprehend the mathematical structures that lead to the correct approach. This would constitute a considerable change in current instructional practices on this critical topic. Van Dooren et al. (2010) eloquently posit that "if we want students to come to grips with the differences between additive and proportional situations, it may be beneficial to scrutinize and systematically redesign all examples and exercises related to proportional reasoning that students are confronted with" (p. 378).

The algebraic concept of linearity is engrained in students because of extensive focus and experience with linear equations at the middle school level. This is not usually problematic in most proportional situations, but lack of experience and instruction on nonlinear situations results in students' tending to see the relationship between length and area as linear rather than quadratic, and the relationship between length and volume as linear rather than cubic. As a result, students apply the linear scale factor instead of its square for area or its cube for volume to determine the impact on an enlarged or reduced figure. As an example, students would likely answer that doubling the sides of a cube would result in doubling the volume rather than resulting in 8 times the volume. Modestou and Gagatsis (2007) studied this issue and concluded:

> Linearity is a knowledge which is successful in a particular context and for a particular set of situations. However, its application outside that context results in false responses accompanied by a strong belief in their correctness. These responses are recurrent and seem to be quite universal and resistant to a variety of forms of support aimed at overcoming the problem (p. 80).

Because students persistently see the linear function everywhere, educators must break the pattern of linearity and ensure that the multidimensional impact of increase or decrease is included as an integral component of proportional-reasoning instruction.

Role of Language and Symbolism

Although not a specific math concept, a theme essential to mathematics understanding is the role played by mathematics language and symbolism. Language plays a dual role not only in describing or defining mathematics content but in using instructional language in the teaching process. Converging with the themes of fractions and proportional reasoning, Dunston and Tyminski (2013) theorize that “mathematics vocabulary instruction is particularly important in the middle grades because this is when the serious development of the language of mathematics begins and when mathematical learning focuses on numbers’ multiplicative structures and relationships” (p. 40). Dunston and Tyminski also propose that developing mathematics vocabulary knowledge is integral to students’ development of abstract reasoning ability, which enables them to progress beyond operations to problem solving.

There are terms in mathematics, such as *integer, radius, trapezoid,* and *quotient,* that have specific mathematical definitions and pose no confusion with standard English. However, there are words that may already exist in students’ vocabulary that have one meaning in standard English but a totally different definition in mathematics. Such terms as *degree, difference, mean, product, obtuse, root,* and *similar* are but a few examples of terms with distinct meanings in mathematics and in standard English. In all likelihood, those who have been educated in the American system will not recollect teachers’ focusing on clarification of such terms with dual meanings. The language used in traditional instruction also can cause confusion with the use of such terms as *reduce* or *cancel* and phrases such as *goes into*. Mathematics instruction typically does not focus on these language issues but assumes that students absorb and understand both the vocabulary and the instructional language. However, Dunston and Tyminski (2013) indicate that “the positive effects of vocabulary instruction on students’ learning are well documented, and the ability to speak the language of mathematics is essential to understanding” (p. 44). A study by Cirino, Tolar, Fuchs, & Huston-Warren (2016) found “direct effects of language on proportional reasoning, which was consistent with other studies that identified linguistic skills as crucial for solving problems involving proportional reasoning” (p. 113). Findings from this type of research place an emphasis on a more deliberate focus on the language of mathematics.

Unlike most other disciplines, the language of mathematics includes symbolism and visual representation as means of written communication. The language and symbolism of mathematics are components of mathematics and mathematics instruction that are in need of increased attention and focus. Evidence from Boyer et al. (2008) indicates that children’s proportional-reasoning abilities are directly affected by the representations they are given. Difficulties with mathematical symbols have been well documented for learning both fractions and algebra (Booth & Newton, 2012). Ni and Zhou (2005) assert that assigning meaning to different representations is particularly critical when children are learning about fractions, and conclude that “a common observation is that for otherwise similar fraction tasks, children would show contrasting performances between those involving symbols and those not involving the symbols” (p. 28). Teachers must specifically address even symbols such as the equal sign, which could be far less understood than one would assume. For example, in their research, Booth and Newton found that “middle school students who held a naïve view of the equal sign performed worse on equation-solving tasks compared to students who understood the sign as an equivalence relation” (p. 247).

Other Factors

Although other mathematics content topics emerged from the research that focused on identification of problematic concepts that hinder the transition to higher mathematics, inclusion of all topics would be an overwhelming task. Topics such as slope, word problems, solving and graphing equations, and so forth, are important concepts or processes in mathematics progression and deserve the attention of middle school educators. However, limiting the scope of the research to a few topics was necessary in order for the practitioners to focus on and address a smaller number of key issues realistically and effectively.

Other related findings are worthy of mention because of their potential impact on student success. One example is the hierarchical nature of mathematics. The learning of most mathematical topics involves a certain amount of prerequisite knowledge. It is possible that students may struggle as much from lack of prerequisite knowledge as the intricacy or complexity of the new topic being learned. For example, Cirino et al. (2016) found that “facility with basic facts and multistep whole-number operations aids all later mathematics because these are often needed at some point in order to solve fractions, proportions, and complex word problems” (p. 97). Such research highlights the progressive development of mathematics knowledge along a procedural or conceptual continuum, which in turn, necessitates that teachers focus on the prerequisite expertise necessary to master a new concept. An additional implication can be found through Seethaler, Fuchs, Star, and Bryant’s study (2011) citing that students’ incoming skills “underscore the importance of intervening early to address students' deficits with foundational mathematics skills to offset future and more pervasive difficulty” (p. 541).

A fairly familiar aspect of American culture is an embedded belief according to which many parents and students believe that mastery of mathematics is restricted to a privileged few. This obstacle to success has been studied and is typically manifested in comparison with Asian education systems. As one example, Galeshi (2014) found that, compared with eighth-grade Chinese Taipei students, American students lacked basic skills such as proportion, word-problem skills, and basic algebraic knowledge. A plausible explanation by Galeshi was the cultural difference between Asian and American parents and teachers. American parents and teachers believed that students’ mathematical performance depended on inherent ability and innate intelligence rather than hard work and persistence, while Asian parents and teachers believed that mathematical performance primarily depended on effort and hard work. Real efforts at the community, district, school, and classroom levels to reverse that American perspective could possibly help improve student performance.

Research Question 2. What Core Knowledge Do Teachers Need to Have About Mathematics in Order to Have Students Successfully Make the Critical Transitions to Higher Level Mathematics?

To improve mathematics achievement, educators have focused attention on quality curricula and developing content standards, but “no curriculum teaches itself, and standards do not operate independently of professionals’ use of them” (Ball, Hill, & Bass, 2005, p. 14). Consequently, our education system is primarily dependent on qualified teachers who understand the subject matter in these standards and curricula. The way teachers teach is a critical component of the education process, but the quality of math instruction is equally dependent on teachers’ content expertise. Teachers cannot teach what they do not know and can only teach to the depth of their understanding. The types of mathematics content that students need in order to transition to more complex mathematics has been investigated here, but this then leads to the related question: “What core knowledge do teachers need to have about mathematics in order to have students successfully make those critical transitions to higher mathematics?”

According to Ball, Hill, et al. (2005), studies consistently reveal that the mathematical knowledge of many teachers is dismayingly thin” (p. 14). The authors add that one factor is that teachers are products of the same system they are trying to improve, a system in which “their own opportunities to learn mathematics have been uneven, and often inadequate, just like those of their non-teaching peers” (p. 14). Before delving into the content knowledge that is essential, it is important to note that reform efforts have emphasized the idea of conceptual understanding of mathematics. Hurst and Hurrell (2016) describe conceptual understanding as robust knowledge that includes knowledge of what a mathematical concept or process is, how those connect to other concepts, and why processes work as they do. Traditional mathematics has focused on fluent computation, but according to Hurst and Hurrell, “to maximize the effectiveness of procedural fluency, it must be underpinned by conceptual understanding” (p. 35).

Certain mathematics concepts are fundamental to students’ learning and can cause a domino effect of misunderstanding if not taught well. If teachers have only a superficial knowledge of mathematics that is based on rote memorization and procedures, they may lead their students to think of mathematics as a series of mindless rules and formulas devoid of deep thought and reasoning. Ball and Forzani (2011) assert that “if teachers are inattentive to important aspects of the ideas that they teach, students may develop misconceptions or distorted understandings of key concepts—many of which may interfere with the pursuit of more demanding learning goals later” (p. 20). Research by Masters (2012) resulted in a similar conclusion: “If a teacher has a thin understanding of a concept, how to teach that concept, or how a student might conceive of that concept, this can lead to the creation or enhancement of students’ underdeveloped or flawed reasoning” (p. 2).

Weak teacher content knowledge also influences mathematics instruction. According to Hill, Ball, and Schilling (2008), “the quality of the modifications made to curriculum materials, the goals for student learning, and even beliefs about what mathematics *is* were shaped by teachers’ knowledge” (p. 497). A study by Ojose (2014) found that “teachers’ knowledge of content does significantly affect the way and manner instruction is delivered” (p. 41). This study investigated teachers’ understanding of two axioms in algebra and found that 50% of the participating teachers did not possess the content knowledge needed to teach those axioms. A key conclusion of the Ojose study was that “with regard to connections between content knowledge and pedagogy, while teachers with content knowledge were able to describe teaching enactments in convincing, innovative, and sometimes fun ways, teachers with no knowledge offered skeletal explanations loaded with routines and rituals consistent with traditional instructional practices” (p. 41).

Link to Fractions

Earlier, the concept of fractions was noted as a fundamental mathematics topic critical to success in secondary mathematics. For these successful transitions to occur, a strong foundation of fractions and fraction computation must occur at the elementary school level. Siegler et al. (2012) assert that “one likely reason for students’ limited mastery of fractions and division is that many U.S. teachers lack a firm conceptual understanding of fractions and division” (p. 696). In a related study, Siegler and Lortie-Forgues (2015) found that many U.S. teachers have a weak conceptual understanding of computation with fractions, which curtails the understanding that they, in turn, can transfer to students. Instead of an emphasis on different algorithms to execute operations on fractions, “teachers should place more emphasis on the conceptual understanding of fractions” (Charalambous & Pitta-Pantazi, 2007, p. 311). However, this would be a difficult undertaking if the teachers themselves lacked that conceptual understanding of fractions.

Rayner, Pitsolantis, and Osana (2009) conducted a study to determine the relationship between preservice teachers’ anxiety levels and their knowledge of fractions, “a topic in the elementary mathematics curriculum that is notorious for the challenges it provides teachers" (p. 66). The authors did find that preservice teachers with high levels of mathematics anxiety did encounter difficulties in solving fraction problems and that those with low levels of mathematics anxiety exhibited a high conceptual understanding of fractions. These same authors suggest that teachers may fear what they don't understand, which in turn, can have the effect that "teachers who are anxious about mathematics may spend less time in the classroom teaching the subject and may impart negative attitudes about it to their students" (p. 61). As an unintended consequence, this can then result in students' lacking the strong foundation in fractions that is necessary for future success.

Link to Proportional Reasoning

The transition from additive reasoning to multiple or proportional reasoning also was noted as critical for student success at the middle and high school levels. Masters (2012) found that studies exploring teachers' proportional-reasoning expertise indicated that middle school teachers often lacked a deep understanding of proportional reasoning. The analysis of the student proportional-reasoning data in the Masters study suggests that students whose teachers have lower levels of knowledge perform at lower levels on proportional reasoning problems, and this analysis also shows that teachers' weak expertise in proportional reasoning leads to a focus on the application of procedures such as cross-multiplication instead of conceptual understanding. According to Masters, "some teachers harbor the same misunderstandings and misconceptions related to proportional reasoning that have been thoroughly documented amongst students" (p. 3). As an illustration, in a typical nonproportional problem that students mistook to be proportional, the Masters data showed that 71% of teachers in the study made that same mistake.

Link to Mathematical Language

The critical role of language and symbolism was previously noted. Ball, Hill, et al. (2005) asserted that an emergent theme in their research was the centrality of mathematical language and the need for a special kind of teachers' fluency with mathematical terms. In their data, the authors repeatedly saw "the need for teachers to have a specialized fluency with mathematical language, with what counts as a mathematical explanation, and with how to use symbols with care" (p. 21). In addition, expertise in the use of mathematical language and symbolism goes far beyond understanding meanings. Ball, Hill, et al. note that "teachers must constantly make judgments about how to define terms and whether to permit informal language or introduce and use technical vocabulary, grammar, and syntax" (p. 21). Teachers, especially at the elementary level, must constantly determine when imprecise, informal language would be pedagogically preferable and when that type of vernacular should be replaced by more formal mathematical terminology that would better develop correct understanding.

Beyond Knowledge of Specific Content Topics

Ball, Hill, et al. (2005) assert that "effective teaching entails a knowledge of mathematics above and beyond what a mathematically literate adult learns in grade school, a liberal arts program, or even a career in another mathematically intensive profession, such as accounting or engineering" (p. 45). However, the research revealed additional facets of content expertise of teachers. According to Hill, Rowan, and Ball (2005), teacher's content knowledge has traditionally been measured using variables such as courses taken, results from subject matter tests, or degrees attained. However, findings such as those by Hill, Blunk, et al. (2008) indicate that prospective teachers who seem knowledgeable in upper level mathematics have significant difficulties in teaching tasks such as explaining division of fractions to students. Just because a teacher understands the way to find the perimeter or area, this is no guarantee that the teacher can analyze the validity of a student's assertion about the relationship between perimeter and area (Ball, Thames, & Phelps, 2008).

Although the content knowledge of a teacher is critical, effectiveness in teaching is also dependent on factors that affect the way content knowledge is utilized in classrooms. According to Hill, Rowan, et al. (2005), teachers' content expertise will only help students learn mathematics if the teachers are able to use this content knowledge to perform teaching tasks such as listening to and guiding student conversations, selecting and making use of good assignments, developing appropriate assessments, and understanding student thinking. Hill, Schilling, and Ball (2004) contend that "rather than focusing simply on how much mathematics an individual knows, we must also ask how that knowledge is held and used by the individual—whether they can use their mathematical knowledge to generate representations, interpret student work, or analyze student mistakes" (p. 28).

Approximately 3 decades ago, Shulman (1986) introduced the notion of "pedagogical content knowledge," which referred to the special nature of the subject matter knowledge required for teaching (p. 9). This concept serves as the bridge between general pedagogical knowledge and general knowledge of subject matter. According to Hill, Schilling, et al. (2004), "the concept of pedagogical content knowledge includes familiarity with topics children find interesting or difficult, the representations most useful for teaching a specific content idea, and learners' typical errors and misconceptions" (p. 5). Shulman's notion of pedagogical content knowledge is a critical component in understanding the demands of teaching, but it is general in nature, and more investigation is necessary to apply this idea to mathematics instruction. An understanding of content is important for effective instruction, but what constitutes understanding of the content is not clearly defined (Ball, Thames, et al., 2008). Examining content and its role in instruction is an area that has historically received little attention—to the extent that this has been dubbed the “missing paradigm in research on teaching and teacher knowledge” (Ball, Thames, et al., p. 390). Although new research has focused directly on teacher content knowledge, this research is also seeking to identify the ways in which “content knowledge for teaching is distinct from disciplinary content knowledge” (Ball, Thames, et al., p. 392).

Mathematical Knowledge for Teaching

A new area of knowledge that goes beyond teacher content expertise has been referred to as “mathematical knowledge for teaching.” According to Hill, Blunk, et al. (2008), this concept refers to

> not only the mathematical knowledge common to individuals working in diverse professions, but also the subject matter knowledge that supports that teaching, for example, why and how specific mathematical procedures work, how best to define a mathematical term for a particular grade level, and the types of errors students are likely to make with particular content (p. 431).

Ball, Thames, et al. (2008) add that mathematical knowledge for teaching is more about teaching than about teachers, so the focus is on actions and processes involved in mathematics instruction and the mathematical demands of those tasks. According to Ball, Thames, et al., “this places the emphasis on the use of knowledge in and for teaching rather than on teachers themselves” (p. 394). However, teachers are still an integral component because mathematical knowledge for teaching involves “knowledge of mathematical ideas, skills of mathematical reasoning, fluency with examples and terms, and thoughtfulness about the nature of mathematical proficiency” (Ball, Thames, et al., p. 395).

An integral component of instruction intertwined with mathematical knowledge for teaching is expertise with respect to the way students reason about, know, or learn mathematics content. According to Hill, Ball, et al. (2008), this knowledge of content and students “is used in tasks of teaching that involve attending to both the specific content and something particular about learners, for instance, how students typically learn to add fractions and the mistakes or misconceptions that commonly arise during this process” (p. 375). Through experience, teachers see that students often make certain errors on particular topics or processes, discover what topics are likely to be difficult, and learn that some strategies or explanations work better than others. However, Hill, Ball, et al. (2008) assert that "although most scholars, teachers, and teacher educators would agree that teachers' knowledge of students' thinking in particular domains is likely to matter, what constitutes such knowledge has yet to be understood" (p. 395). Hill, Ball, et al. (2008) add that "although teachers' knowledge of students' mathematical thinking and learning is widely believed to be an important component of teacher knowledge, it remains underspecified, and its relationship to student achievement undemonstrated" (p. 396).

Research on the concepts of mathematics knowledge for teaching and knowledge of students' interaction with content suggests that there is knowledge used in classrooms beyond formal subject matter knowledge. However, the field has made little progress in developing a coherent theoretical framework for these ideas about content knowledge for teaching, and this has resulted in a situation in which "these ideas remain theoretically scattered and lacking clear definition" (Ball, Thames, et al., 2008, p. 394). In addition, even when a grounded theory about the elements and definition of mathematical knowledge for teaching is developed, there is still the issue of demonstrating, or proving, that improving this knowledge also enhances student achievement (Ball, Hill, et al., 2005). The idea of mathematical knowledge for teaching is a vexing issue that warrants additional investigation for an understanding of the complexity and depth of this concept.

Addressing the Teacher Content Knowledge Issue

The research indicates that there is a body of mathematical knowledge for teaching that goes beyond expertise in mathematics content. Although the research did identify specific topics, such as fractions and proportional reasoning, that are critical foundations for success in mathematics, it is apparent that only focusing on improving teachers' expertise in these topics would be insufficient. Efforts by state departments of education (SDEs) to establish a teacher corps with strong expertise in mathematics knowledge for teaching must focus on both the preparation of new teachers and professional development (PD) for teachers already in the classroom. Ball, Hill, et al. (2005) contend that there is a place in professional preparation for concentrating on teachers' specialized, applied mathematical knowledge unique to the work of teaching. That being the case, SDEs must coordinate with institutions of higher learning to ensure that teacher preparation programs change to include the ideas brought out by this research.

Any PD programs geared to teachers already in mathematics classrooms must be developed and implemented with care. It is possible that well-designed PD (e.g., intensive mathematics content courses accompanied by follow-up workshops) implemented with fidelity can bolster teachers' knowledge and improve the mathematical quality of instruction. Research such as that of Hill, Blunk, et al. (2008) suggests that professional development cuts both ways in that poor-quality or inadequately implemented PD can produce dismal results. Hill, Blunk, et al. (2008) noted countless examples of supposedly "new" mathematical methods that were implemented without meaning or purpose; "teachers returned to class with mathematical tasks they themselves did not understand, and then taught them to students as procedures instead of non-routine, complex problems to be solved" (p. 500). SDEs also must be aware that shortcomings in PD are not limited to the classroom level.

For example, Rittle-Johnson and Jordan (2016) noted that there is large variability in teacher and principal commitment to change when districts mandate teacher PD. Rittle-Johnson and Jordan found that "even though their teacher professional development for sixth-grade teachers on three topics (fractions, ratios and proportions, and expressions and equations) was mandated, school administrators often co-opted professional development sessions for other purposes, leading to substantially less professional development time than intended" (p. 30). This indicates the critical need for monitoring and review to ensure effective implementation and evaluation of whatever PD is designed or selected.

Although teacher experience and student demographics are not a central theme in this literature review, many studies indicate that inexperienced teachers tend to be assigned to schools with high populations of minority students, which leaves these students with less knowledgeable teachers, who are unable to contribute effectively to their knowledge. As an example, Ball, Hill, et al. (2005) suggest that "a portion of the achievement gap on the National Assessment of Educational Progress and other standardized assessments might result from teachers with less mathematical knowledge teaching more disadvantaged students" (p. 45). In their study, Ball, Hill, et al. found that "the size of the effect of teachers' mathematical knowledge for teaching was comparable to the size of the effect of socioeconomic status on student gain scores. This suggests that improving teachers' knowledge may be one way to stall the widening of the achievement gap as poor children move through school" (p. 44). The research of Ball, Hill, et al. suggests that "one important contribution we can make toward social justice is to ensure that every student has a teacher who comes to the classroom equipped with the mathematical knowledge needed for teaching" (p. 44). These findings have implications for school district policies and decisions regarding the assignment of teachers.

Research Question 3. What Are Best Practices That Teachers Should Use to Teach Mathematics—Specifically, Concepts Related to Higher Level Mathematics?

To address Research Question 3, the project team focused its search on evidence-based or effective mathematics instruction. Twenty-seven articles were selected for the literature review regarding effective teaching practices. Of those, 21 were deemed relevant for the literature review. Among the articles selected, 10 consisted of meta-analyses and research summaries on effective practices. These articles provided a contextual background on the topic. Eleven were either research or evaluative studies on interventions and tools. As literature was screened and later reviewed for themes, two overarching themes emerged: standards-based reform and mathematics learning difficulties. Over the past 25 years, standards-based reform has had an influence on teaching and learning, bringing increased attention to effective instruction. However, standards-based reform is limited in its ability to serve students with disabilities, as efforts in instruction, curriculum, and assessment for students with disabilities may vary from that for the general student population (McDonnell, McLaughlin, & Morison, 1997). In the past 10 years, a growing body of research related to standards-based curriculum and instruction, and mathematics learning difficulties, has provided context to the knowledge base.

Standards-Based Education Reform

More than 25 years ago, former President George H. W. Bush held an education summit at the University of Virginia. While the educational goals that emerged from the event[2] were not realized, standards-based accountability became a tool of measure for federal and state governments for more than 30 years. Each administration following that of George H. W. Bush proposed educational initiatives that reinforced the value of standards-based accountability (Klein, 2014), from America 2000 to Goals 2000 to No Child Left Behind and Race to the Top. In 2007, the Council of Chief State School Officers convened to discuss the creation of state standards.

In 2010, Alabama adopted the Alabama College and Career Ready Standards (CCRS) (Alabama State Department of Education, 2016). The adoption of CCRS has not only shaped the way the state prepares students for postsecondary success but has informed teachers' instructional

[2] In 1991, the U.S. Department of Education unveiled "America 2000." The proposal, a collaboration of the National Governors Association, the U.S. Congress, and the White House, was composed of six voluntary national education goals.

practice. As a standards-based reform, CCRS aligns with instruction and assessment. Among the research articles that were screened, a number referred to the intervention as standards based.

Mathematics Learning Disabilities

The second overarching theme that emerged from the literature search was mathematics learning disabilities (MLD). To understand MLD, *learning disability* (LD) must be defined. The knowledge base on LD reveals that LD is a "discrepancy between a student's academic achievement and their apparent capacity to learn (Lyon, 1996, p. 56)."

> The Individuals with Disabilities Education Act (IDEA) defines a learning disability as follows: "The child does not achieve adequately for the child's age or to meet State-approved grade-level standards in one or more of the following areas, when provided with learning experiences and instruction appropriate for the child's age or State-approved grade-level standards: oral expression; listening comprehension; written expression; basic reading skills; reading fluency skills; reading comprehension; and mathematics calculation." [3]

The research on learning disabilities initially centered on reading learning disabilities (RLD). In a meta-analysis, the authors report that between 1996 and 2005, the ratio of RLD studies to MLD studies was 5:1, an improvement from the previous decade of 16:1 (Gersten et al., 2009). Since 2005, the National Center for Special Education Research within the Institute of Education Sciences (IES) has funded research and research training projects. In the same span of time, federal funding for research on special education topics increased (Jayanthi, Gersten, & Baker, 2008). Branches of research on LD include the nature of LD and instructional intervention.

Explicit and Systematic Instruction

As articulated under Research Question 1, mathematical concepts, processes, properties, and language and symbolism are connected and interdependent to one another. Practitioners and researchers who study MLD contend that, to support students' mastery in concepts like word-problem solving and operations, instruction must be explicit and systematic (National Mathematics Advisory Panel, 2008; Gersten et al., 2009; Shin & Bryant, 2015). Explicit and systematic instruction consists of (a) the teacher's providing students with a clear model for solving a problem, (b) an opportunity for the student to practice the newly learned strategy,

[3] 34 U.S.C. § 300.8 (2007)

(c) an opportunity for the student to think aloud, and (d) extensive feedback from the teacher (National Mathematics Advisory Panel, 2008).

The literature review yielded three studies on two interventions that met WWC standards,[4] which include explicit and systematic mathematics instruction. Please be advised that interventions identified are not to be considered endorsements of a proprietary product; rather, they offer examples of effective practices. In addition, while the level of evidence on explicit and systematic mathematics instruction is strong, not all the themes or components highlighted yield the same effect when isolated from other instructional components involved in the interventions referenced.

Building Whole-Number Skills

The ROOTS Intervention is an early-learning intervention to support students' foundational skills. Clarke et al. (2018) evaluated ROOTS, a multitier system of support on whole-number concepts and skills for kindergartens with mathematics learning difficulties. The curriculum includes 50 lessons, offering interventionists scripted guidelines that are reflective of effective and systematic instruction. ROOTS is a Tier 2 intervention, and thus, interventionists worked with children in small groups to address their challenges with whole numbers. The researchers highlighted the ROOTS adoption of an explicit and systematic framework, noting the following:

- Modeling: Interventionists overtly count objects to determine how many are in a countable set
- Practice: Structured learning opportunities to reinforce concepts and skills
- Verbalization: Students must verbalize their thinking when solving whole-number problems
- Feedback: Students receive informational feedback to correct mistakes (Clarke et al., 2018)

Schema-Based Strategy

Schema-based instruction allows students to gain word-problem skills by grouping them into types and using problem–solution rules to solve for each problem type (Fuchs et al., 2008; Jitendra et al., 1998). Jitendra et al. evaluated a schema-based strategy in which students also

[4] The Institute of Education Sciences at the U.S. Department of Education manages the What Works Clearinghouse (WWC), which reviews research on programs, products, practices, and policies in education. To access the recent WWC standards, visit https://ies.ed.gov/ncee/wwc/handbooks#procedures

received Tier 2 supports. Students were in small groups of three to six students for 30 to 45 minutes each session, for 17 to 20 days. The schema involved explicit and systematic instruction. The schema strategy involves the following:

- Modeling: Teachers instruct students on ways to identify word problem types;
- Practice: Students review the word problems;
- Verbalization: Through facilitative questions, students identify the word-problem types; and
- Feedback: Students receiving explicit feedback and additional modeling (Jitendra et al., 1998).

Instruction on Solving Word Problems

> If a teacher has a thin understanding of a concept, how to teach that concept, or how a student might conceive of that concept, this can lead to the creation or enhancement of students' underdeveloped or flawed reasoning (Masters, 2012).

Research Question 2 examines the core knowledge necessary for teachers. The literature revealed the significance of teachers' concept understanding to building their students' understanding and use of mathematical concepts and skills. Current research on MLD reveals a strong level of evidence on interventions that include instruction on the underlying structures of word problems (Gersten et al., 2009). Students' difficulties with word problems also can be coupled with difficulties in reading; therefore, researchers have examined approaches to developing students' word-problem skills (Fuchs et al., 2008; Jitendra et al., 1998). By identifying the structures of word problems, students build procedural fluency from conceptual understanding (National Council of Teachers of Mathematics, 2014). The literature review yielded three studies on two interventions that included solving word problems.

Schema-Broadening Tutoring

Similar to the schema-based strategy, students received preventive tutoring three times per week, 30 minutes per session, for 12 weeks. To support students with MLD, Fuchs et al. (2008) broadened schema-based instruction to include explicit and systematic instruction through preventive tutoring. This approach taught students to do the following: (a) understand the mathematical structure of the problem type, (b) recognize the basic schema for a problem type, (c) solve the problem type, and (d) transfer solution methods to word problems (Fuchs et al., 2008).

Computer-Based Instruction

An intervention introduced in later grades, Cognitive Tutor Algebra I, is designed to support students' understanding of algebraic concepts in Grades 7–12. Cognitive Tutor involves a blended curriculum in which 40% of students' classroom time involves use of computer adaptive software; however, teachers are trained to integrate technology into classroom instruction (Ritter, Kulikowich, Lei, McGuire, & Morgan, 2007). In a study by Empirical Education, researchers reported that teachers received a research-based pedagogy, and students received multiple representations of word problems, just-in-time feedback, a skillometer, and a blended curriculum of computer lab and classroom activities that complement each other (Cabalo, Ma, & Jaciw, 2007).

Facilitate Meaningful Mathematical Discourse

In two recent meta-analyses of mathematics instruction, student verbalization, or discourse, was shown to strengthen students' mathematical reasoning skills (Gersten et al., 2009; Shin & Bryant, 2015). When teachers use explicit and systematic mathematics instruction, students have an opportunity to think aloud, shifting from memorization to building their concept understanding by verbalizing the way to solve word problems (Fuchs et al., 2008; Gersten et al., 2009; Jitendra et al., 1998).

In the interventions presented, mathematics instruction included student verbalization, either with the teacher or among peers. For example, in the ROOTS intervention, the interventionists paired students to solve whole-number word problems. While in pairs, the kindergarten students articulated the strategy they used to solve for the problem and the way they arrived at the final answer. As the students discussed their strategies and answers, the interventionist listened in on conversations to determine the students' grasp of the strategy taught and provided corrective feedback, further extending concept understanding (Clarke et al., 2018). Schema-based strategy also involved student verbalization. Teachers engaged in frequent exchanges with students to reinforce the schema for identifying word problem types and solving word problems. Although Cognitive Tutor Algebra I is computer based, 60% of classroom activities involve instruction. The program includes collaboration and student presentations; students also reveal their work in peer review (Ritter et al., 2007; What Works Clearinghouse, 2016).

Conclusion

The project team conducted three rounds of review, starting out with 79 pieces on mathematics, primarily published between 2008 and 2018. In Rounds 2 and 3, the team re-reviewed the literature to address the research questions of interest to ALSDE:

1. What specific knowledge do students need to have to make the transitions to higher level mathematics?
2. What core knowledge do teachers need to have about mathematics in order to have students successfully make those critical transitions to higher level mathematics?
3. What are best practices that teachers should use to teach mathematics, specifically concepts related to higher level mathematics?

Themes

Fractions, proportional reasoning, language and symbolism, building mathematical knowledge for teaching, teacher and student use of mathematical knowledge, teacher content knowledge, explicit and systematic instruction, instruction on solving word problems, facilitation of meaningful mathematical discourse

Themes emerged as the literature was reviewed. For Research Questions 1 and 2, fractions, proportional reasoning, and mathematical language were significant to building upon higher level mathematics. The literature also revealed that both students and teachers lacked the conceptual understanding necessary for mastery (Ball, Hill, et al., 2005; Pelen & Artut, 2016; Shin & Bryant, 2015; Siegler et al., 2015). For Research Question 3, the literature revealed a growing body of research on mathematics learning disabilities. From the research, effective approaches included explicit and systematic instruction, instruction on solving word problems, and facilitating meaningful mathematical discourse. The literature revealed that effective instruction involved a clear explanation of the model that students could follow and replicate (Clarke et al., 2018). The literature also suggested that teachers provide students with the underlying structures for solving problems, thereby teaching students to understand the way to identify the types of word problems they are being presented with (Fuchs et al., 2008; Jitendra et al., 1998). Finally, research found that it was helpful for students to verbalize their thinking with teachers and peers (Clarke et al., 2018; Fuchs et al., 2008; Jitendra et al., 1998; Ritter et al., 2007). Even though the themes are presented by research question, they are not to be considered mutually exclusive. They are inextricably intertwined, with each supporting and impacting the others, as they offer guidance on what is necessary for effectively teaching mathematics.

Implications

In April 2017, the Alabama Mathematics Strategic Planning Committee, which consisted of five subcommittees (i.e., Access, Equity, Empowerment, and Advocacy; Curriculum and Instruction; Community and Workforce Development; Professional Development; and Teacher Education Programs in Higher Education), released 83 recommendations, in response to the state's slow rate of growth in mathematics achievement. While the literature review was not available prior to the planning committee's work, the emergent themes from the knowledge base do reinforce a number of the recommendations presented to ALSDE. For example, the Curriculum and Instruction subcommittee recommended providing Ongoing Assessment Project (OGAP) training for every teacher in K–8 in the following areas and grade levels: additive reasoning (Grades K–3), multiplicative reasoning (Grades 3–5), fraction reasoning (Grades 3–8), and ratio/proportional reasoning (Grades 5–8). According to the Consortium for Policy Research in Education (2018), OGAP "trains teachers to use single or multiple math items of high cognitive demand to gather information on student thinking and then analyze that information using frameworks based on research on student thinking in mathematics" (Consortium for Policy Research in Education). This recommendation not only mirrors the findings on what students should know (i.e., fraction reasoning and proportional reasoning) in order to make the transition into higher level mathematical concepts but also endorses the need for teachers to improve their mathematical knowledge.

Another recommendation that reflects the findings is from the Access, Equity, Empowerment, and Advocacy subcommittee. The subcommittee's very first recommendation was to do the following:

> Improve school accountability for the use of the mathematical teaching practices (National Council of Teachers of Mathematics, 2014) determined to increase each and every students' engagement and achievement in mathematics and embedding the standards for mathematical practice from the Alabama Course of Study (2016) in their instruction through the adoption and creation of teacher and student assessments that highlight these standards (Alabama State Department of Education, 2017, p. 8).

With an emphasis on each and every student, the subcommittee recognized the various ways in which teachers engaged and built students' conceptual understanding of mathematics. Even though explicit and systematic instruction is not among the practices, this practice also reinforces the need to both build and strengthen students' understanding of mathematical procedures.

With a large number of recommendations to work from, the project team attempted to provide current literature that enhanced and deepened the department's understanding of what teachers should know and be able to do in order to effectively teach mathematics. While there were examples of where the literature review and the recommendations reflected similar points, the literature review also identified a potential gap between the recommendations and the research base. The burgeoning knowledge base on mathematics learning difficulties suggests the value of Response to Intervention (RTI) in mathematics. The project team noted that among the recommendations for curriculum and instruction, the state's use of RTI was not highlighted. The knowledge base on RTI is mostly associated with reading but does have a benefit for mathematics instruction, particularly in early grades, and should be recognized (VanDerHeyden, n.d.). As ALSDE considers next steps for mathematics achievement, the current knowledge base on student and teacher content knowledge and effective instructional practice present additional findings from which to choose and to incorporate into the department's efforts to improve mathematics education.

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Appendix A. Methodology and Screening Process

Methodology and Screening Process

The Alabama State Department of Education (ALSDE) requested a literature review in an effort to inform the recommendations presented by the Alabama Strategic Mathematics Planning Committee (the committee). ALSDE developed the research questions that guided the search, and the Southeast Comprehensive Center (SECC) provided its initiatives designed to increase the proficiency of all K–8 students in mathematics and science. As an initial step, ALSDE and SECC formed two project teams for each content area. Both groups conducted a literature search to obtain the latest theoretical frameworks and empirical research on pedagogy and practice for educators, along with students' academic, thinking, and reasoning skills. The project team selected for the literature review on mathematics (Grades K–8), consists of senior technical assistance consultants, content experts, and researchers, including ALSDE staff. Prior to the search, the project team developed a systematic approach to conducting the literature review beginning with the development of research questions, key terms, and the process for cataloguing literature.

The sources include the Education Resources Information Center (ERIC) and other federally funded databases and organizations, research institutions, academic research databases, and general Internet search engines. For details, please see the Search of Databases and Websites section at the end of this report. SECC and ALSDE vetted the references as a step in assessing the knowledge base on mathematics in Grades K–8. The references in the report are from the most commonly used resources of research but are not necessarily comprehensive; other relevant references and resources may exist.

Keywords and Search Strings Proposed by Research Team

Research Question 1

- "mathematics"
- "rational numbers"
- "fractions"
- "proportional or multiplicative reasoning"
- "absolute or relative reasoning"
- "word problems
- "integers"
- "operations"
- "TIMSS"

- “NAEP”
- “student performance”
- “multi-step”

Research Question 2

- “best practices AND mathematics AND elementary education”
- “best practices AND mathematics AND middle school or junior high”
- “whole numbers AND mathematics instruction AND elementary education”
- “whole numbers AND mathematics instruction AND middle school or junior high”
- “rational numbers AND mathematics instruction AND elementary education”
- “rational numbers AND mathematics instruction AND middle school or junior high”
- “number sense AND mathematics instruction AND elementary education”
- “number sense AND mathematics instruction AND middle school or junior high”
- “algebraic reasoning AND mathematics instruction AND elementary education”
- “algebraic reasoning AND mathematics instruction AND middle school or junior high”
- “absolute reasoning AND mathematics instruction AND elementary education”
- “absolute reasoning AND mathematics instruction AND middle school or junior high”
- “proportional reasoning AND mathematics instruction AND elementary education”
- “proportional reasoning AND mathematics instruction AND middle school or junior high”
- “integers AND mathematics instruction AND elementary education”
- “integers AND mathematics instruction AND middle school or junior high”

Research Question 3

- “mathematics pedagogy”
- “content knowledge AND mathematics”
- “curricular knowledge AND mathematics”
- “teaching and learning mathematics”
- “mathematics pedagogy AND elementary education”
- “mathematics knowledge”
- “teaching and learning mathematics AND elementary education”
- “mathematics pedagogy AND elementary or middle school or junior high”
- “teaching and learning mathematics AND middle school or junior high”

Search of Databases and Websites

Institute of Education Sciences Sources: Regional Educational Laboratory Program, What Works Clearinghouse, National Center for Education Statistics, Institute of Education Sciences (IES), IES Practice Guides

Other Federally Funded Sites: Center on Innovation and Improvement, Center on Instruction, National Comprehensive Center for Teacher Quality, Common Core of Data, National Center for Research on Early Childhood Education

Additional Data Resources: Education Development Center, ERIC, EBSCO databases, JSTOR database, FirstSearch (OCLC), ProQuest, Educator's Reference Complete, Google Scholar, Google, general Internet search

Criteria for Inclusion

When the mathematics review team examined articles, they considered—among other things—four factors:

- Date of the publication: The most current information is included, except in the case of nationally known seminal resources.
- Source and funder of the report/study/brief/article: Priority is given to IES, nationally funded, and certain other vetted sources known for strict attention to research protocols.
- Methodology: Randomized controlled trial studies, surveys, self-assessments, literature reviews, policy briefs. Priority for inclusion generally is given to randomized controlled trial study findings, but the reader should note at least the following factors when basing decisions on these resources: numbers of participants (just a few? thousands?); selection (did the participants volunteer for the studies or were they chosen?); representation (were findings generalized from a homogeneous or a diverse pool of participants? was the study sample representative of the population as a whole?).
- Existing knowledge base: Although we strive to include vetted resources, there are times when the research base is slim or nonexistent. In these cases, we have included the best resources we could find, which may include newspaper articles, interviews with content specialists, organization websites, and so on.

Appendix B. Thematic Synthesis

Thematic Synthesis

1. Content area teams will meet to discuss findings from literature. Meetings will aid writing teams in identifying potential themes to develop for synthesis.
    a. Key findings documented in the literature review matrix:
        i. Note the extent to which key findings emerged from other sources and how strong the evidence is to support the key findings.
    b. Consider common themes that emerged from literature:
        i. Revisit categories to consider themes: publication type, type of evidence/evaluation/research, description of intervention, unit of analysis, and key findings.
    c. Highlight literature that writing teams should revisit or review for an understanding of potential themes.
        i. Make updates to literature matrix:
            1. Confirm whether the marking of literature as relevant (i.e., column G) has changed after reading.
            2. Confirm whether research questions assigned to literature have changed.
    d. Discuss whether the literature addresses the research questions.
2. Writing team will meet separately to discuss outcome of content area team meeting and begin thematic synthesis.
    a. In preparation for synthesis, writing teams will combine references into a single matrix for review.
        i. The writing team will capture descriptive information from the matrix, including total number of relevant articles, by the following:
            1. Publication type
            2. Type of evidence/evaluation/research
            3. Research question
    b. Review the key findings documented by content area team members to confirm potential themes that emerged during meeting with larger group.

i. Discuss whether key findings support potential themes. The writing team should revisit the extent to which key findings emerged from other sources and how strong the evidence is to support each key finding.

1. Split themes identified.
2. Begin to organize or categorize literature into themes.
3. Review or reread literature to expand on emergent theme.
4. Draft write-up of theme.

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at American Institutes for Research